Electronic Control Systems

http://www.ent.mrt.ac.lk/~rohan/teaching/EN2142/EN2142.html

Ref Book: Classical Control Systems, ISBN 978-81-8487-194-4

Prof. Rohan Munasinghe
Department of Electronic and Telecommunication Engineering
Faculty of Engineering
University of Moratuwa 10400

Contents

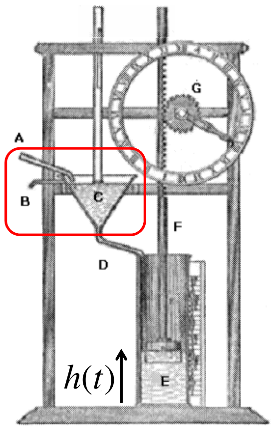
1. Historical Control Systems : "**Feedback by Design**"
2. Mathematical Modeling of plants/systems
   - Input output relationship as an ordinary differential equation (ODE)
3. Determining the response of the plant/system
   - Poles, theoretical solution (Laplace)
   - Simulation (MatLab) + performance verification
4. Design a feedback control system
5. Frequency domain analysis of Control systems
   - Bode plots (Mag/Phase response)
6. PID: Most famous industrial controller
7. Controller implementation: Analog (OPAmp)
8. Digital Control
   1. Digital Redesign (of analog controller)
   2. Digital implementation (of analog controller)

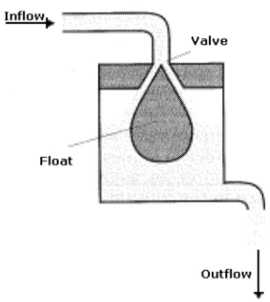
1. Historical Control Systems: "feedback by design"

- 250 BC flow regulated water clock
  - Ctesibius, a Greek Inventor

Constant flow rate

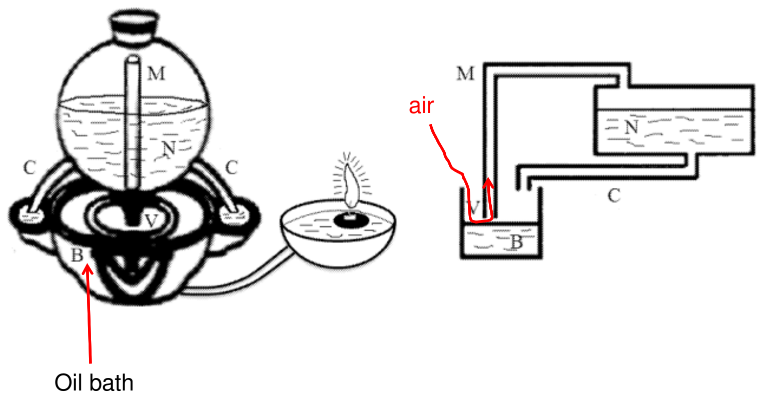
$$h(t)=\frac{V(t)}{A}=\frac{Ft}{A}=Kt$$





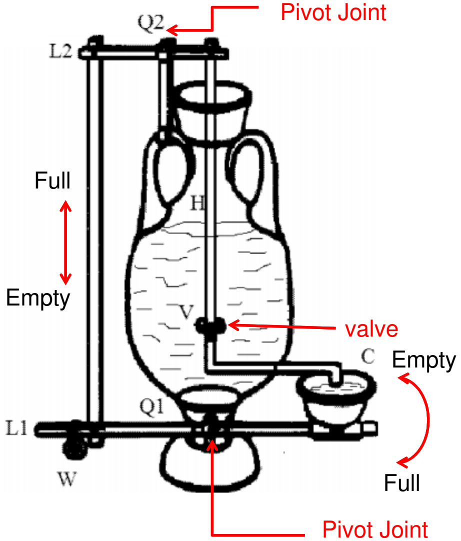
1. Historical Control Systems:

- Self Re-filling mechanism (200BC)
  - Philon, a Greek inventor



1. Historical Control Systems:

- Weight Regulated Liquid Filling Device(1st Century AD)



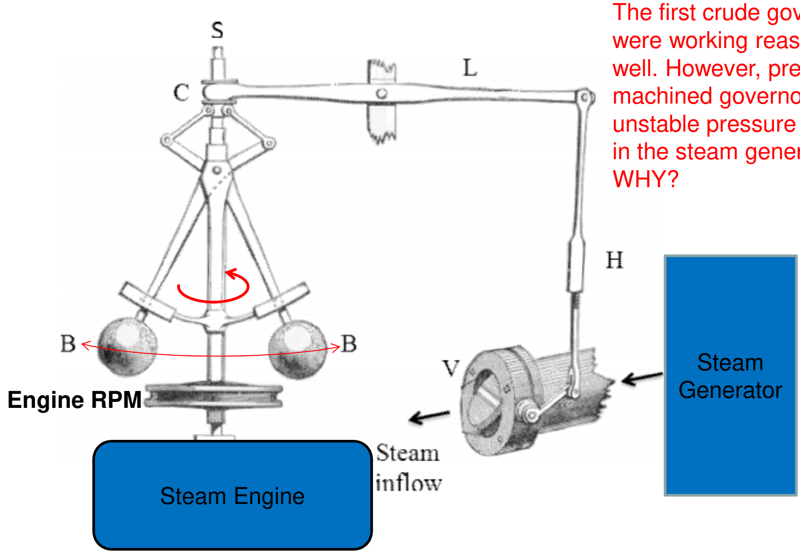
Close (default) Position

Open Position

1. Historical Control Systems:

- Flyball Governor (James Watt 1788)

The first crude governors were working reasonably well. However, precisely machined governors showed unstable pressure oscillations in the steam generators. WHY?



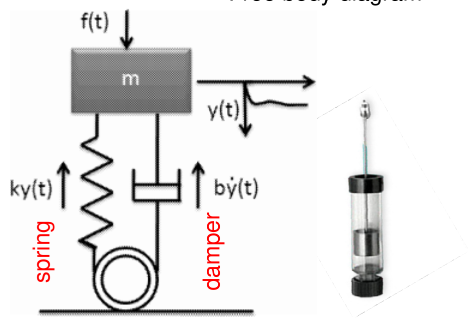
2. Mathematical Modeling of Plant/System

Prof. Rohan Munasinghe
Department of Electronic and Telecommunication Engineering
Faculty of Engineering
University of Moratuwa 10400

Plant Model: Mechanical System

- Shock absorber

Free body diagram



- Spring resists displacement (reactive force is prop to displacement)
- Damper resists speed (reactive force is proportional to speed)

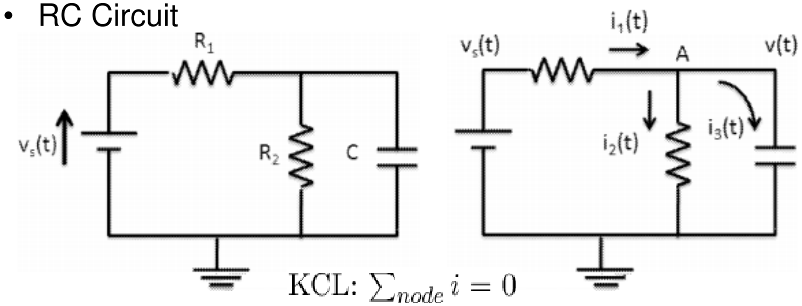
$$m\ddot{y}(t) = f - ky(t) - b\dot{y}(t) \quad (2.1)$$

$$\ddot{y}(t) + \frac{b}{m}\dot{y}(t) + \frac{k}{m}y(t) = \frac{1}{m}f(t) \quad (2.2)$$

Second order model

Plant Model: Electrical System

- RC Circuit



$$\text{KCL: } \sum_{node} i = 0$$

$$\begin{aligned} i_2(t) + i_3(t) &= i_1(t) \\ C\dot{v}(t) + \frac{1}{R_2}v(t) &= \frac{v_s(t) - v(t)}{R_1} \\ R_1R_2C\dot{v}(t) + R_1v(t) &= R_2(v_s(t) - v(t)) \\ R_1R_2C\dot{v}(t) + (R_1 + R_2)v(t) &= R_2v_s(t) \end{aligned}$$

First order model

Plant Model: Electrical System

$$\begin{aligned} \dot{v}(t) + \frac{R_1 + R_2}{R_1R_2C}v(t) &= \frac{1}{R_1C}v_s(t) \\ \dot{v}(t) + av(t) &= bv_s(t) \end{aligned} \qquad (2.3)$$

$$a = \frac{R_1+R_2}{R_1R_2C} \qquad b = \frac{1}{R_1C}$$

What have we learned so far?

- System models are ordinary differential equations (ODEs).
- System complexity is represented by the model order
- The response (output) of the plant can be obtained by solving the model ODE for a given forcing function (input)
- Laplace transforms can be used to solve ODEs efficiently

Laplace Transforms

function	$f(t)$	$F(s)$
unit impulse	$\delta(t)$	1
unit step	$u(t)$	$\frac{1}{s}$
time exponent	e^{at}	$\frac{1}{s-a}$
cosin	$\cos\omega t$	$\frac{s}{s^2+\omega^2}$
power of time	t^n	$\frac{n!}{s^{n+1}}$
linearity	$\alpha_1 f_1(t) \pm \alpha_2 f_2(t)$	$\alpha_1 F_1(s) \pm \alpha_2 f_2(s)$
exponential scaling	$e^{at}f(t)$	$F(s-a)$
time shift	$f(t \pm T)$	$e^{\pm sT}F(s)$
time multiplication	$tf(t)$	$-\frac{d}{ds}F(s)$
differential	$\frac{d^n f(t)}{dt^n}$	$s^nF(s) - s^{n-1}f(0) - s^{n-2}f'(0) \cdots - f^{n-1}(0)$
integral	$\int f(t)dt$	$\frac{1}{s}F(s)$
time scaling	$f(at)$	$\frac{1}{a}e^{\frac{1}{a}}F(s)$
convolution integral	$\int_0^t f(\tau)g(t-\tau)d\tau$	$F(s)G(s)$

System Response with Laplace

- RC Circuit Model $\quad \dot{v}(t) + av(t) = bv_s(t) \quad$ (2.3)
  - When transformed into Laplace domain $\mathcal{L}\{\ \}$ - forward transformation

For DC Voltage

$$v_s(t) = Vu_s(t)$$

$$\begin{aligned} sV(s) - v(0) + aV(s) &= bV\frac{1}{s} \\ (s+a)V(s) &= v(0) + bV\frac{1}{s} \\ V(s) &= \frac{1}{s+a}v(0) + \frac{b}{s(s+a)}V \end{aligned} \qquad (3.47)$$

Partial fraction $\quad V(s) = \frac{1}{s+a}v(0) + \frac{b}{a}\left(\frac{1}{s} - \frac{1}{s+a}\right)V$

- Response (Method 1: Partial Fraction)
  - When transformed back to time domain $\mathcal{L}^{-1}\{\ \}$ - inverse transformation

$$v(t) = v(0)e^{-at} + \frac{b}{a}V\left(1 - e^{-at}\right) \qquad (3.49)$$

System Response with Laplace

Response (Method 2: Convolution Integral)

(3.47), $$V(s) = \frac{1}{s+a}v(0) + \frac{b}{s(s+a)}V$$

$$L^{-1}\left\{\frac{1}{s}\right\} = u_s(t) \qquad L^{-1}\left\{\frac{1}{s+a}\right\} = e^{-at}$$

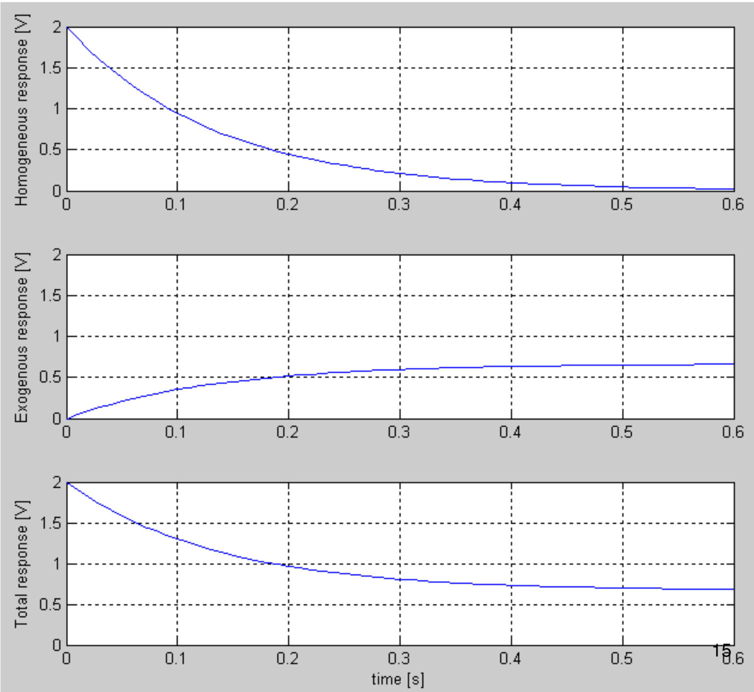
- Transforming back to time domain

$$\begin{aligned} v(t) &= v(0)e^{-at} + bV_s\int_0^t e^{-a\tau}u_s(t-\tau)d\tau \\ &= v(0)e^{-at} + bV_s\int_0^t e^{-a\tau}.1d\tau \\ &= v(0)e^{-at} + \frac{-b}{a}V_s e^{-a\tau}|_0^t \\ &= \underbrace{v(0)e^{-at}}_{\text{Homogeneous Response}} + \underbrace{\frac{b}{a}V_s\left(1-e^{-at}\right)}_{\text{Exogenous Response}} \end{aligned} \quad (3.50)$$

Matlab Simulation: RC Circuit

```
%% First Order Response: RC Circuit
R1=1000; R2=2000; C=200*10^-6;          % circuit components
a=(R1+R2)/(R1*R2*C); b=1/(R1*C);        % model coefficients
v0=2;                                   % initial condition
dur=0.6                                 % simulation duration
t=[0:0.01:dur];

%% Homogeneous Response
yH=v0*exp(-a*t);

%% Exogenous Response
yE=(b/a)*(1-exp(-a*t));

%% Total Response
yT=yH+yE;

%% Plot graphs
subplot(311); plot(t,yH); axis([0 dur 0 2]);
ylabel('Homogeneous response [V]'); grid on;

subplot(312); plot(t,yE); axis([0 dur 0 2]);
ylabel('Exogenous response [V]'); grid on;

subplot(313); plot(t,yT); axis([0 dur 0 2]);
ylabel('Total response [V]'); xlabel('time [s]'); grid on;
```


Matlab Simulation: Step Response



Observations and Conclusions

- Homogeneous response (Initial condition response) dies out leaving no remaining response in the long run. The time period where this response lasts is known as transient response.
- Exogenous response is what remains in the long run after transient time. This response finally remains as the steady state response.
- In order to understand the input output relationship, homogeneous response has to be removed.

System Response with Laplace

- Mechanical System Response

(2.2) $$\ddot{y}(t) + \frac{b}{m}\dot{y}(t) + \frac{k}{m}y(t) = \frac{1}{m}f(t)$$

$$\ddot{y}(t) + 2\sigma\dot{y}(t) + \rho y(t) = \eta f(t) \quad (3.51)$$

Where

$$2\sigma = \frac{b}{m},\ \rho = \frac{k}{m},\ \text{and } \eta = \frac{1}{m}$$

- Transforming into Laplace domain

$$\begin{aligned} s^2Y(s) - sy(0) - y'(0) + 2\sigma[sY(s) - y(0)] + \rho Y(s) &= \eta F(s) \\ (s^2 + 2\sigma s + \rho)Y(s) - y(0)s - [2\sigma y(0) + y'(0)] &= \eta F(s) \end{aligned}$$

$$Y(s) = \frac{y(0)s + [2\sigma y(0) + y'(0)]}{(s^2 + 2\sigma s + \rho)} + \frac{\eta}{(s^2 + 2\sigma s + \rho)}F(s) \quad (3.52)$$

- There exist three different responses based on the determinant of the denominator polynomial (characteristic equation)

$$(2\sigma)^2 - 4\times 1\times\rho = 4(\sigma^2 - \rho) \longleftarrow \Delta(s) = s^2 + 2\sigma s + \rho = 0$$

System Response: Partial Fractions

- *Case 1:* $\sigma^2 - \rho > 0\ (b > 2\sqrt{mk})$

$K_1 = y(0)$, and $K_2 = 2\sigma y(0) + y'(0)$

(3.52) $$Y(s) = \frac{K_1 s + K_2}{(s-\alpha_1)(s-\alpha_2)} + \frac{\eta}{(s-\alpha_1)(s-\alpha_2)}F(s) \longleftarrow F(s) = \frac{A}{s}$$

where $\alpha_1, \alpha_2 = -\sigma \pm \sqrt{\sigma^2 - \rho}$ **negative, real, distinct poles**

Poles are determined by the system parameters

Case 1: Partial Fractioning *Using coverup method (see Appendix)*

$$\begin{aligned} Y(s) &= \frac{K_1 s + K_2}{(s-\alpha_1)(s-\alpha_2)} + \frac{\eta}{(s-\alpha_1)(s-\alpha_2)}\frac{A}{s} \quad (3.55) \\ &= \frac{P_1}{(s-\alpha_1)} + \frac{P_2}{(s-\alpha_2)} + \eta A\left(\frac{Q_1}{(s-\alpha_1)} + \frac{Q_2}{(s-\alpha_2)} + \frac{Q_3}{s}\right) \end{aligned}$$

where $$\underbrace{P_1 = \frac{K_1\alpha_1 + K_2}{\alpha_1 - \alpha_2}, P_2 = \frac{K_1\alpha_2 + K_2}{\alpha_2 - \alpha_1}}_{\text{initial conditions and system parameters}}, \underbrace{Q_1 = \frac{1}{\alpha_1(\alpha_1 - \alpha_2)}, Q_2 = \frac{1}{\alpha_2(\alpha_2 - \alpha_1)}, Q_3 = \frac{1}{\alpha_1\alpha_2}}_{\text{system parameters}}$$

System Response: Partial Fractions

- *Transforming back to time domain*

$$\begin{aligned} y(t) &= P_1 e^{\alpha_1 t} + P_2 e^{\alpha_2 t} + \eta A\left(Q_1 e^{\alpha_1 t} + Q_2 e^{\alpha_2 t}\right) + \eta A Q_3 \\ &= \underbrace{\eta A Q_3}_{\text{Steady-state response}} + \underbrace{(P_1 + \eta A Q_1)e^{\alpha_1 t} + (P_2 + \eta A Q_2)e^{\alpha_2 t}}_{\text{Transient response}} \quad (3.56) \end{aligned}$$

Decay with time 'cos $\propto_1 \propto_2$ are -ve

System Response: Convolution Integral

- *Case 1: Convolution Integral Method*

(3.55), $$\begin{aligned} Y(s) &= \left\{\frac{P_1}{(s-\alpha_1)} + \frac{P_2}{(s-\alpha_2)}\right\} + \eta\left\{\frac{P_3}{(s-\alpha_1)} + \frac{P_4}{(s-\alpha_2)}\right\}F(s) \\ &= \frac{P_1}{(s-\alpha_1)} + \frac{P_2}{(s-\alpha_2)} + \eta\frac{P_3}{(s-\alpha_1)}F(s) + \eta\frac{P_4}{(s-\alpha_2)}F(s) \quad (3.57) \end{aligned}$$

where $P_3 = \frac{1}{\alpha_1 - \alpha_2}$ $P_4 = \frac{1}{\alpha_2 - \alpha_1}$

System Response with Laplace

- *Transforming back to time domain*

$$y(t) = P_1 e^{\alpha_1 t} + P_2 e^{\alpha_2 t} + \eta P_3 \int_0^t e^{\alpha_1 (t-\tau)} f(\tau) d\tau + \eta P_4 \int_0^t e^{\alpha_2 (t-\tau)} f(\tau) d\tau$$

- *For* $f(t) = A u_s(t)$

$$\begin{aligned}
y(t) &= P_1 e^{\alpha_1 t} + P_2 e^{\alpha_2 t} + \eta P_3 \int_0^t e^{\alpha_1 (t-\tau)} A d\tau + \eta P_4 \int_0^t e^{\alpha_2 (t-\tau)} A d\tau \\
&= P_1 e^{\alpha_1 t} + P_2 e^{\alpha_2 t} + \eta P_3 e^{\alpha_1 t} \int_0^t e^{-\alpha_1 \tau} A d\tau + \eta P_4 e^{\alpha_2 t} \int_0^t e^{-\alpha_2 \tau} A d\tau \\
&= P_1 e^{\alpha_1 t} + P_2 e^{\alpha_2 t} - \frac{\eta P_3 A e^{\alpha_1 t}}{\alpha_1} e^{-\alpha_1 \tau}|_0^t - \frac{\eta P_4 A e^{\alpha_2 t}}{\alpha_2} e^{-\alpha_2 \tau}|_0^t \\
&= P_1 e^{\alpha_1 t} + P_2 e^{\alpha_2 t} - \frac{\eta P_3 A e^{\alpha_1 t}}{\alpha_1} (e^{-\alpha_1 t} - 1) - \frac{\eta P_4 e^{\alpha_2 t} A}{\alpha_2} (e^{-\alpha_2 t} - 1) \\
&= P_1 e^{\alpha_1 t} + P_2 e^{\alpha_2 t} + \frac{\eta P_3 A}{\alpha_1} (1 - e^{\alpha_1 t}) + \frac{\eta P_4 A}{\alpha_2} (1 - e^{\alpha_2 t}) \\
&= \underbrace{-\eta A \left(\frac{P_3}{\alpha_1} + \frac{P_4}{\alpha_2} \right)}_{\text{Steady-state response}} + \underbrace{\left(P_1 + \frac{\eta A P_3}{\alpha_1} \right) e^{\alpha_1 t} + \left(P_2 + \frac{\eta A P_4}{\alpha_2} \right) e^{\alpha_2 t}}_{\text{Transient response}}
\end{aligned} \quad (3.59)$$

Steady State and Transient Responses

- Transient response
  - Decaying response
  - Depends both on Initial conditions and external forcing function

$$\left(P_1 + \frac{\eta A P_3}{\alpha_1} \right) e^{\alpha_1 t} + \left(P_2 + \frac{\eta A P_4}{\alpha_2} \right) e^{\alpha_2 t}$$

- Steady State Response
  - The sustainable response
  - Depends on external forcing function

$$-\eta A \left(\frac{P_3}{\alpha_1} + \frac{P_4}{\alpha_2} \right)$$

Simulation: Case 1: Over Damped Shock-Absorber

- System parameters k=125N/cm b=700Ns/cm m=50 kg
  - Strong damper
  - Speed is severely resisted
- Then, from (3.51) σ=7, ρ=2.5, and η=0.02
- Consequently, the two system poles are

α_1=-0.181 and α_2=-13.819

-ve real distinct poles

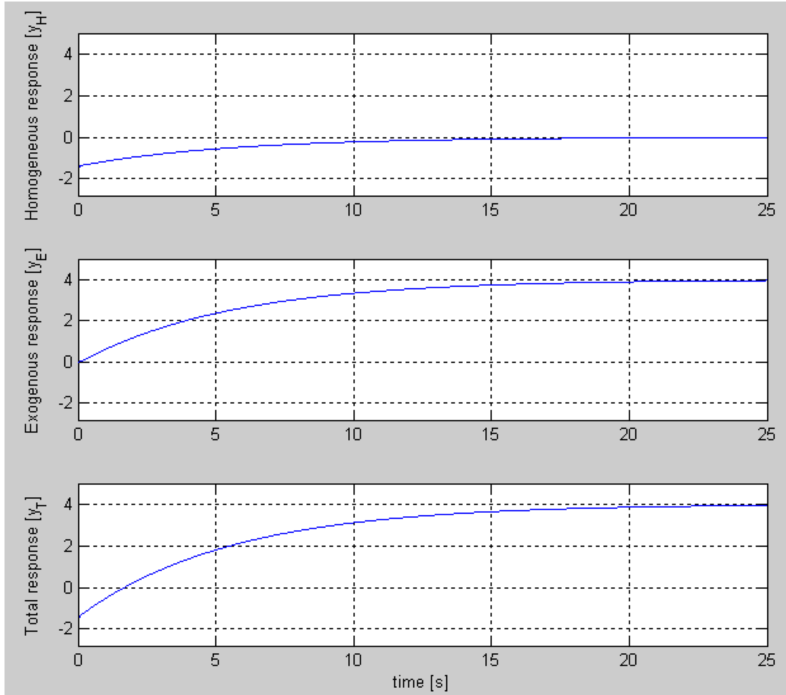
Matlab Code

```
%% Shock3p8 : Second Order Response of Shock-Absorber
dur=25;m=50;            % weight of the rider
b=700; k=125;           % case 1: b[Ns/cm] k[N/cm] m[kg] over damped
%b=700; k=b^2/(4*m)      % case 2: b[Ns/cm] k[N/cm] m[kg]  critically damped
%b=300; k=2450           % case 3: b[Ns/cm] k[N/cm] m[kg] under damped

sigma=b/(2*m), rho=k/m, eta=1/m      % model coefficients
d=sigma^2-rho                        % determinant
A=10*m;                              % weight step input

t=[0:0.01:dur];
y0=-1.50; yd0=1.80;                  % Initial conditions
k1=y0; k2=2*sigma*y0+yd0;

%% Determination of poles
if  d>0
    alpha1=-sigma+sqrt(d)
    alpha2=-sigma-sqrt(d)
    p1=(alpha1*k1+k2)/(alpha1-alpha2);
    p2=(alpha2*k1+k2)/(alpha2-alpha1);
    q1=1/(alpha1*(alpha1-alpha2));
    q2=1/(alpha2*(alpha2-alpha1));
    q3=1/(alpha1*alpha2);
    yH=p1*exp(alpha1*t)+p2*exp(alpha2*t);
    yE=eta*A*(q1*exp(alpha1*t)+q2*exp(alpha2*t)+q3);
elseif  d==0
```

Matlab Code

```
      alpha=-sigma
      p5=(k1*alpha+k2)/alpha; p6=-k2/alpha; p7=1/alpha; p8=-1/alpha;
      yH=(p6+p5)*exp(alpha*t)+alpha*t.*exp(alpha*t);
      yE=eta*A*(-p7*t.*exp(alpha*t)+p8*(exp(alpha*t)-1)/alpha);
else
      omega=sqrt(-d), phiH=atan2(k1,(k2-sigma*k1)/omega)
      phiE=atan2(omega,sigma)
      K=sqrt(k1^2+(k2-sigma*k1)^2/omega^2)
      yH=K*exp(-sigma*t).*sin(omega*t+phiH);
      yE1=exp(-sigma*t).*sin(omega*t+phiE);
      yE=eta*A/omega*(omega/(omega^2+sigma^2)-yE1);
end

%% Total Response
yT=yH+yE;

%% Plot graphs
subplot(311); plot(t,yH); axis([0 dur -2.8 5]);
ylabel('Homogeneous response [y_H]'); grid on;
subplot(312); plot(t,yE); axis([0 dur -2.8 5]);
ylabel('Exogenous response [y_E]'); grid on;
subplot(313); plot(t,yT); axis([0 dur -2.8 5]);
ylabel('Total response [y_T]'); xlabel('time [s]'); grid on;
```


Simulation: Over Damped



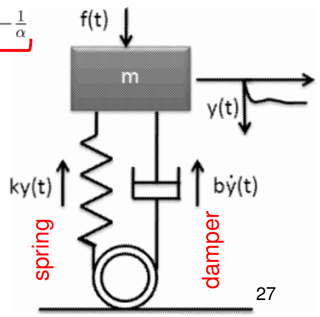
System Response

- *Case 2:* $\sigma^2 - \rho = 0,\ (b = 2\sqrt{mk})$ → Real, -ve coincident poles

(3.52) →
$$Y(s) = \frac{K_1 s + K_2}{(s-\alpha)^2} + \frac{\eta}{(s-\alpha)^2}F(s)$$

$$Y(s) = \left\{\frac{P_5 s}{(s-\alpha)^2} + \frac{P_6}{(s-\alpha)}\right\} + \eta\left\{\frac{P_7 s}{(s-\alpha)^2} + \frac{P_8}{(s-\alpha)}\right\}F(s) \quad (3.61)$$

where $\underbrace{P_5 = \frac{K_1\alpha+K_2}{\alpha},\ P_6 = -\frac{K_2}{\alpha}}_{\text{IC and System}},\ \underbrace{P_7 = \frac{1}{\alpha},\ \text{and}\ P_8 = -\frac{1}{\alpha}}_{\text{System}}$



System Response: *Case 2 Critical Damping*

For $f(t) = Au_s(t) \longrightarrow F(s) = A\frac{1}{s}$

$$\begin{aligned} Y(s) &= \left\{\frac{P_5 s}{(s-\alpha)^2} + \frac{P_6}{(s-\alpha)}\right\} + \eta\left\{\frac{P_7 s}{(s-\alpha)^2} + \frac{P_8}{(s-\alpha)}\right\}A\frac{1}{s} \quad (3.62) \\ &= P_5\frac{s}{(s-\alpha)^2} + P_6\frac{1}{(s-\alpha)}\eta AP_7\frac{1}{(s-\alpha)^2} + \eta AP_8\frac{1}{s(s-\alpha)} \end{aligned}$$

d()/ds 1/s s

$e^{\alpha t}$ $te^{\alpha t}$

$$\frac{d(te^{\alpha t})}{dt} + te^{\alpha t}|_{t=0} = (1+\alpha t)e^{\alpha t} \qquad \int_0^t e^{\alpha t}dt = \frac{1}{\alpha}e^{\alpha t}|_0^t = \frac{1}{\alpha}(e^{\alpha t} - 1)$$

Response

$$\begin{aligned} y(t) &= P_5(1+\alpha t)e^{\alpha t} + P_6 e^{\alpha t} + \eta AP_7 te^{\alpha t} + \eta AP_8\frac{1}{\alpha}(e^{\alpha t} - 1) \\ &= -\frac{\eta AP_8}{\alpha} + \left(\frac{\eta AP_8}{\alpha} + P_6 + P_5\right)e^{\alpha t} + (\eta AP_7 + P_5\alpha)te^{\alpha t} \end{aligned}$$

Simulation: *Case 2 Critical Damping*

Adjust (increase) spring constant to achieve critical damping

earlier

k=125N/cm b=700Ns/cm

k =2450[N/cm] $k = \dfrac{b^2}{2m}$

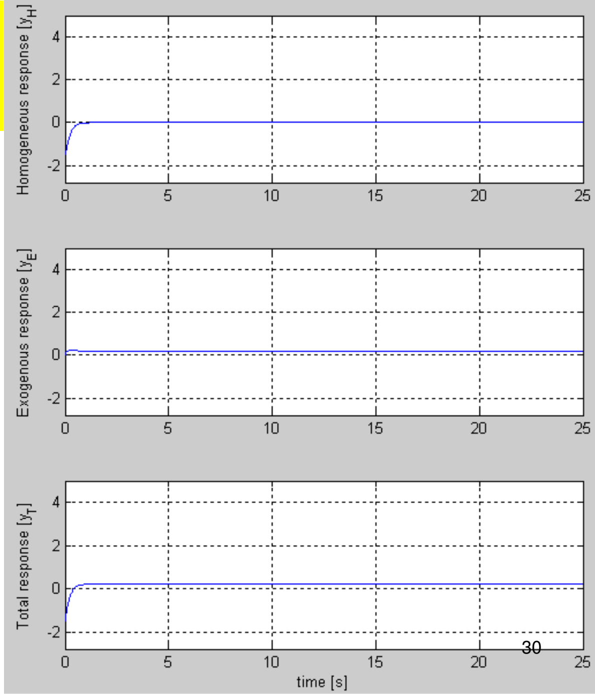
Poles $\alpha_1 = \alpha_2 = -7$

Deflection at steady state $f/k = 500/2450 = 0.2cm$

MatLab Simulation

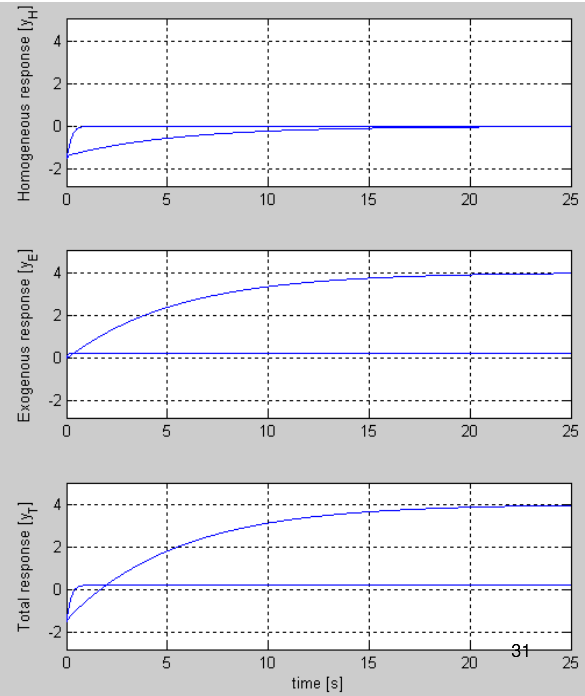
Critical Damping

- Fast response
- No overshoots
- Most energy efficient



Response Comparison

- Over damped response is BIG and slow
- Critically damped response is small and FAST



Response: *Case 3 Under Damped*

- *Case 3:* $\sigma^2 - \rho < 0,\ b < 2\sqrt{km}$ ⇨ Weaker damper Complex Conjugate pair of poles
- *System poles* $\alpha_1, \alpha_2 = -\sigma \pm j\omega$
- *Response*

(3.52),

$$\begin{aligned}
Y(s) &= \frac{y(0)s + [2\sigma y(0) + y'(0)]}{(s+\sigma-j\omega)(s+\sigma+j\omega)} + \frac{\eta}{(s+\sigma-j\omega)(s+\sigma+j\omega)}F(s) \\
&= \frac{K_1 s + K_2}{(s+\sigma)^2 - (j\omega)^2} + \frac{\eta}{(s+\sigma)^2 - (j\omega)^2}F(s) \\
&= K_1\frac{s}{(s+\sigma)^2+\omega^2} + K_2\frac{1}{(s+\sigma)^2+\omega^2} + \eta\frac{1}{(s+\sigma)^2+\omega^2}\frac{A}{s} \\
&= K_1\left[\frac{s+\sigma}{(s+\sigma)^2+\omega^2} - \frac{\sigma}{\omega}\frac{\omega}{(s+\sigma)^2+\omega^2}\right] + \frac{K_2}{\omega}\frac{\omega}{(s+\sigma)^2+\omega^2} \\
&\quad \frac{\eta}{\omega}\frac{\omega}{(s+\sigma)^2+\omega^2}\frac{A}{s}
\end{aligned} \tag{3.65}$$

Response: *Case 3 Under Damped*

but *s* => *(s+σ)* → exponential scaling → $e^{-\sigma t}\cos\omega t$

$\cos\omega t$

$$= K_1\left[\frac{s+\sigma}{(s+\sigma)^2+\omega^2} - \frac{\sigma}{\omega}\frac{\omega}{(s+\sigma)^2+\omega^2}\right] + \frac{K_2}{\omega}\frac{\omega}{(s+\sigma)^2+\omega^2}$$

$$\frac{\eta}{\omega}\frac{\omega}{(s+\sigma)^2+\omega^2}\frac{A}{s} \qquad (3.65)$$

$\sin\omega t$

but *s* => *(s+σ)* → exponential scaling

$e^{-\sigma t}\sin\omega t$

$e^{-\sigma t}\sin\omega t$

$\int_0^t e^{-\sigma t}\sin\omega t dt$

33

Response : Case 3 Under Damped

$$\begin{aligned} y(t) &= K_1 e^{-\sigma t}\cos\omega t - \frac{K_1\sigma}{\omega}e^{-\sigma t}\sin\omega t + \frac{K_2}{\omega}e^{-\sigma t}\sin\omega t \\ &\quad + \frac{\eta A}{\omega}\int_0^t e^{-\sigma t}\sin\omega t dt \\ &= e^{-\sigma t}\left\{K_1\cos\omega t + \frac{(K_2-\sigma K_1)}{\omega}\sin\omega t\right\} \\ &\quad + \frac{\eta A}{\omega}\left\{\frac{\omega}{\omega^2+\sigma^2} - e^{-\sigma t}\sin(\omega t+\phi_E)\right\} \end{aligned}$$

Refer Appendix for derivation

- Decaying sinusoidal indicates an oscillation, which is a result of weaker damper to resist the speed

k=125N/cm b=700Ns/cm Case 1

k =2450[N/cm] Case 2

$f/k = 500/2450 = 0.2cm$

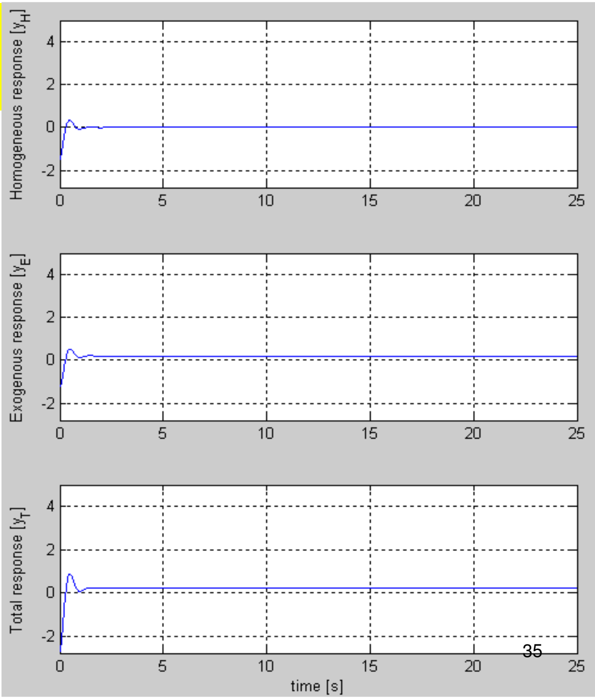
Steady state response

k =2450[N/cm] b = 300 Ns/cm Case 3

34

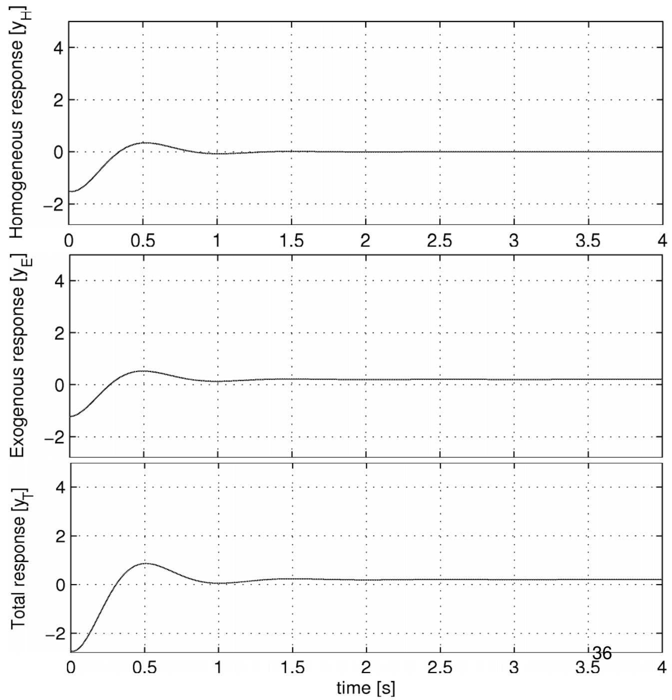
Under Damped Response

- Poles
  $\alpha_1, \alpha_2 = -3 \pm j6.3$
- Oscillatory due to dominant spring action
- Oscillation decays. Response is stable



35

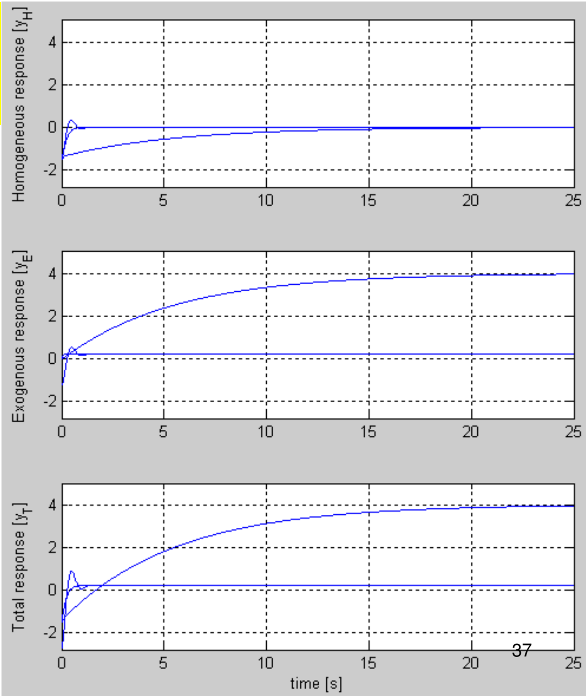
Oscillatory Response



36

Response Comparison

- Critical damped and under damped responses are generally better than over damped response
- Overshoot is generally unacceptable in motion control systems (robots), however, some overshoot is acceptable in process control systems (temperature, pressure)
- Under damped response is the fastest in reaching the level



Poles and Response

Over damped

Critically damped

Under damped